

Theoretical Chemistry and Computational Modelling

Arnout Jozef Ceulemans

Group Theory Applied to Chemistry



 Springer

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ISSN 2214-4714

Theoretical Chemistry and Computational Modelling

ISBN 978-94-007-6862-8

DOI 10.1007/978-94-007-6863-5

Springer Dordrecht Heidelberg New York London

ISSN 2214-4722 (electronic)

ISBN 978-94-007-6863-5 (eBook)

Library of Congress Control Number: 2013948235

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Printed on acid-free paper

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