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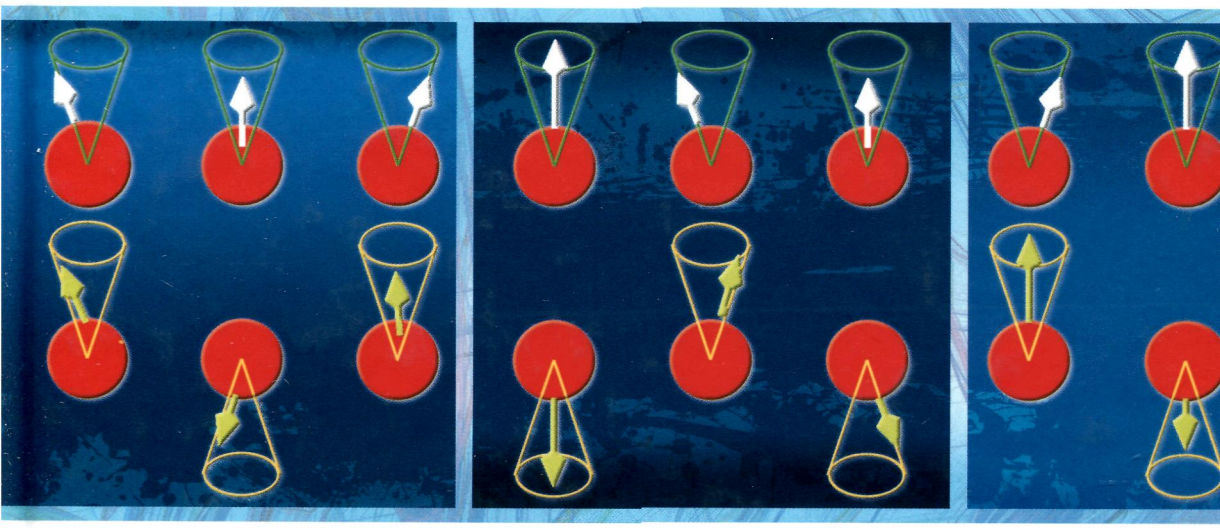
Statistical Mechanics of Magnetic Excitations



From Spin Waves to Stripes and Checkerboards

Enrico Rastelli

World Scientific

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Statistical Mechanics of Magnetic Excitations

From Spin Waves to Stripes and Checkerboards

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World Scientific
NEW JERSEY · LONDON · SINGAPORE · BEIJING · SHANGHAI · HONG KONG · TAIPEI · CHENNAI

Published by

World Scientific Publishing Co. Pte. Ltd.

5 Toh Tuck Link, Singapore 596224

USA office: 27 Warren Street, Suite 401-402, Hackensack, NJ 07601

UK office: 57 Shelton Street, Covent Garden, London WC2H 9HE

British Library Cataloguing-in-Publication Data
A catalogue record for this book is available from the British Library.

Series on Advances in Statistical Mechanics — Vol. 18
STATISTICAL MECHANICS OF MAGNETIC EXCITATIONS
From Spin Waves to Stripes and Checkerboards

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ISBN 978-981-4355-50-6

Typeset by Stallion Press
Email: enquiries@stallionpress.com

Printed in Singapore by B & Jo Enterprise Pte Ltd

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