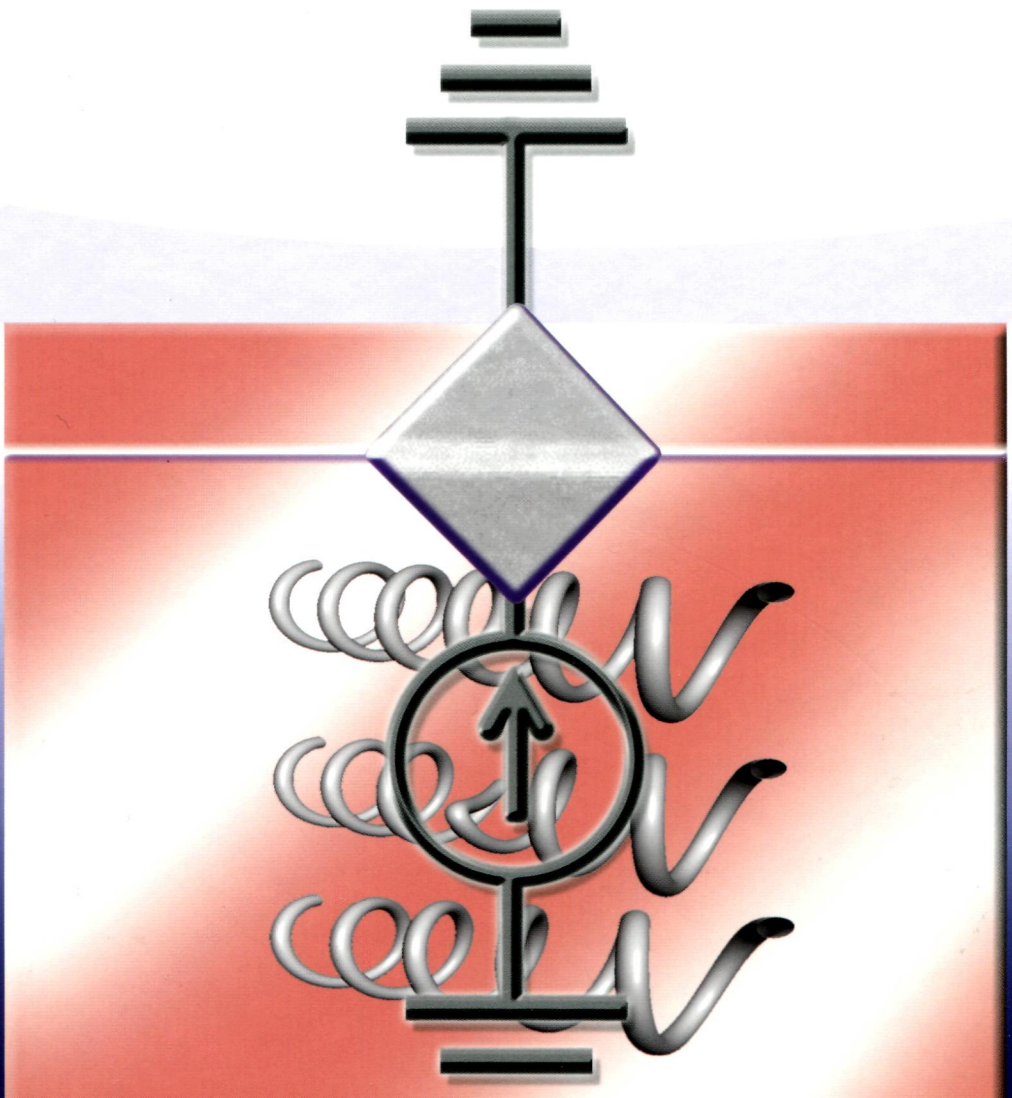


Fabio Altomare, Albert M. Chang

# One-Dimensional Superconductivity in Nanowires



*Fabio Altomare and Albert M. Chang*

# **One-Dimensional Superconductivity in Nanowires**



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## The Authors

### **Dr. Fabio Altomare**

D-Wave Systems Inc.  
100 - 4401 Still Creek Drive  
Burnaby, BC, V5C 6G9  
Canada

### **Prof. Albert M. Chang**

Duke University  
Department of Physics  
Physics Building  
Science Drive  
Durham, NC 27708-0305  
USA

## Cover Figure

Artistic rendering of an 8 nm wide, 20  $\mu\text{m}$  long nanowire fabricated on an InP semiconducting stencil. Electrical contacts to the nanowire (left and right pads) are realized during the nanowire deposition. At the center, a QPS junction is current biased by a current source: The number of windings of the phase of the order parameter decreases because of a phase slippage event. The diagram does not indicate the actual nanowire connection in a circuit. Rendering of the windings courtesy of A. Del Maestro.

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