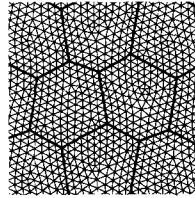


HANDBOOK OF
MICROMECHANICS
AND NANOMECHANICS

edited by Shaofan Li and Xin-Lin Gao





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