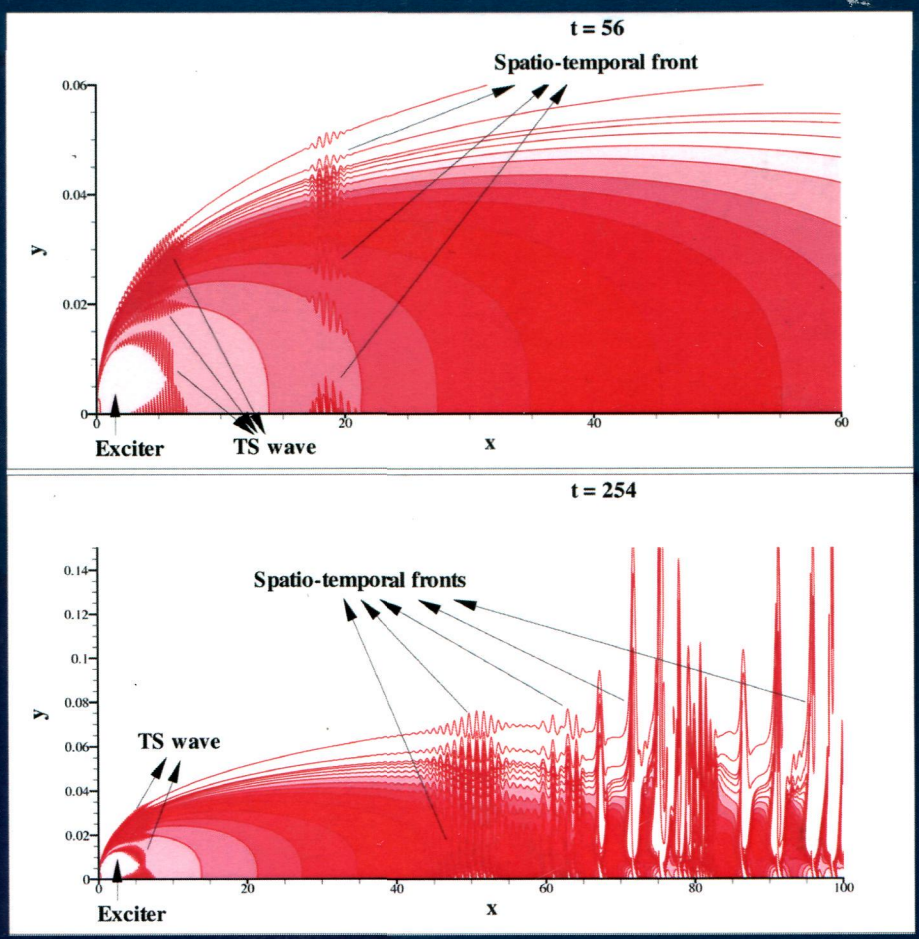
INSTABILITIES OF FLOWS AND TRANSITION TO TURBULENCE



TAPAN K. SENGUPTA

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