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Michael V. Sadovskii

QUANTUM FIELD THEORY

2ND EDITION

TEXTS AND MONOGRAPHS
IN THEORETICAL PHYSICS

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Author
Prof. Dr. Michael V. Sadovskii
Russian Academy of Sciences
Institute for Electrophysics
Amundsenstreet 106
Ekaterinburg 620016
Russia
sadovski@iep.uran.ru

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Contents

Preface — VII

1 Basics of elementary particles — 1
1.1 Fundamental particles — 1
1.1.1 Fermions — 2
1.1.2 Vector bosons — 3
1.2 Fundamental interactions — 5
1.3 The Standard Model and perspectives — 5

2 Lagrange formalism. Symmetries and gauge fields — 9
2.1 Lagrange mechanics of a particle — 9
2.2 Real scalar field. Lagrange equations — 10
2.3 The Noether theorem — 15
2.4 Complex scalar and electromagnetic fields — 20
2.5 Yang–Mills fields — 26
2.6 The geometry of gauge fields — 33
2.7 A realistic example: chromodynamics — 40

3 Canonical quantization, symmetries in quantum field theory — 43
3.1 Photons — 43
3.1.1 Quantization of the electromagnetic field — 43
3.1.2 Remarks on gauge invariance and Bose statistics — 48
3.1.3 Vacuum fluctuations and Casimir effect — 51
3.2 Bosons — 54
3.2.1 Scalar particles — 54
3.2.2 Truly neutral particles — 59
3.2.3 *CPT*-transformations — 62
3.2.4 Vector bosons — 66
3.3 Fermions — 68
3.3.1 Three-dimensional spinors — 68
3.3.2 Spinors of the Lorentz group — 73
3.3.3 The Dirac equation — 80
3.3.4 The algebra of Dirac's matrices — 86
3.3.5 Plane waves — 89
3.3.6 Spin and statistics — 91
3.3.7 *C*, *P*, *T* transformations for fermions — 93
3.3.8 Bilinear forms — 95
3.3.9 The neutrino — 95

4 The Feynman theory of positron and elementary quantum electrodynamics — 103
4.1 Nonrelativistic theory. Green's functions — **103**
4.2 Relativistic theory — **107**
4.3 *Momentum representation* — **111**
4.4 The electron in an external electromagnetic field — **113**
4.5 The two-particle problem — **119**

5 Scattering matrix — 125
5.1 Scattering amplitude — **125**
5.2 Kinematic invariants — **129**
5.3 Unitarity — **132**

6 Invariant perturbation theory — 135
6.1 Schroedinger and Heisenberg representations — **135**
6.2 *Interaction representation* — **136**
6.3 *S*-matrix expansion — **141**
6.4 Feynman diagrams for electron scattering in quantum electrodynamics — **148**
6.5 Feynman diagrams for photon scattering — **153**
6.6 Electron propagator — **156**
6.7 The photon propagator — **159**
6.8 The Wick theorem and general diagram rules — **162**

7 Exact propagators and vertices — 171
7.1 Field operators in the Heisenberg representation and interaction representation — **171**
7.2 The exact propagator of photons — **173**
7.3 The exact propagator of electrons — **179**
7.4 Vertex parts — **183**
7.5 Dyson equations — **186**
7.6 Ward identity — **187**

8 Some applications of quantum electrodynamics — 191
8.1 Electron scattering by static charge: higher-order corrections — **191**
8.2 The Lamb shift and the anomalous magnetic moment — **195**
8.3 Renormalization – how it works — **200**
8.4 "Running" the coupling constant — **204**
8.5 Annihilation of e^+e^- into hadrons. Proof of the existence of quarks — **206**
8.6 The physical conditions for renormalization — **207**
8.7 The classification and elimination of divergences — **211**

8.8 The asymptotic behavior of a photon propagator at large momenta — 215
8.9 Relation between the "bare" and "true" charges — 218
8.10 The renormalization group in QED — 222
8.11 The asymptotic nature of a perturbation series — 225

9 Path integrals and quantum mechanics — 227
9.1 Quantum mechanics and path integrals — 227
9.2 Perturbation theory — 235
9.3 Functional derivatives — 241
9.4 Some properties of functional integrals — 242

10 Functional integrals: scalars and spinors — 247
10.1 The generating functional for scalar fields — 247
10.2 Functional integration — 251
10.3 Free-particle Green's functions — 255
10.4 The Generating functional for interacting fields — 261
10.5 φ^4 theory — 264
10.6 The generating functional for connected diagrams — 271
10.7 Self-energy and vertex functions — 274
10.8 The theory of critical phenomena — 278
10.9 Functional methods for fermions — 292
10.10 Propagators and gauge conditions in QED — 300

11 Functional integrals: gauge fields — 303
11.1 Non-Abelian gauge fields and Faddeev–Popov quantization — 303
11.2 Feynman diagrams for non-Abelian theory — 309

12 The Weinberg–Salam model — 319
12.1 Spontaneous symmetry-breaking and the Goldstone theorem — 319
12.2 Gauge fields and the Higgs phenomenon — 325
12.3 Yang–Mills fields and spontaneous symmetry-breaking — 330
12.4 The Weinberg–Salam model — 337

13 Renormalization — 347
13.1 Divergences in φ^4 — 347
13.2 Dimensional regularization of φ^4-theory — 352
13.3 Renormalization of φ^4-theory — 357
13.4 The renormalization group — 363
13.5 Asymptotic freedom of the Yang–Mills theory — 370
13.6 "Running" coupling constants and the "grand unification" — 377

14 Nonperturbative approaches — 383
14.1 The lattice field theory — **383**
14.2 Effective potential and loop expansion — **395**
14.3 Instantons in quantum mechanics — **400**
14.4 Instantons and the unstable vacuum in field theory — **411**
14.5 The Lipatov asymptotics of a perturbation series — **417**
14.6 The end of the "zero-charge" story? — **419**

Bibliography — 425

Index — 429