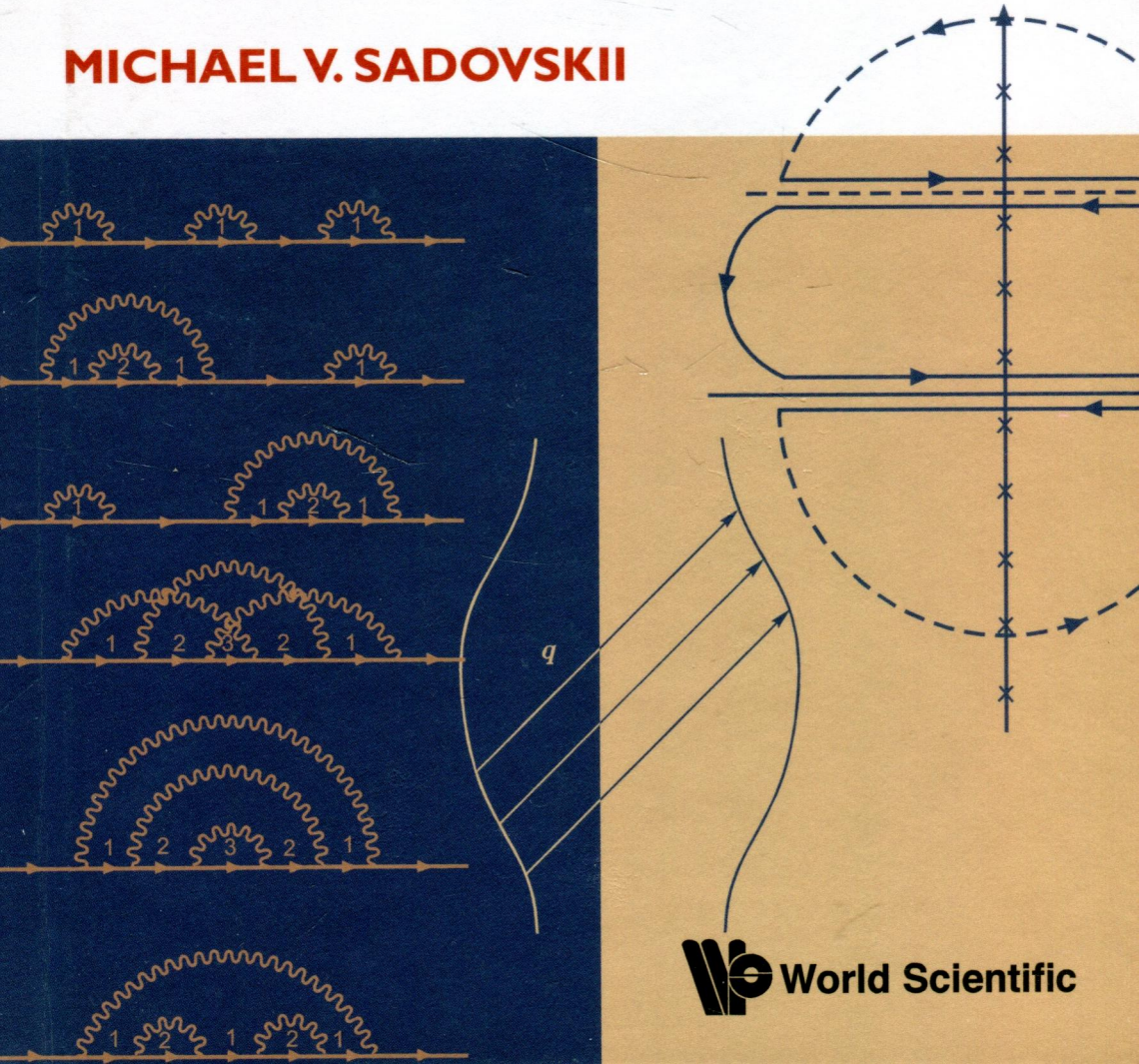
DIAGRAMMATICS

Lectures on Selected Problems in Condensed Matter Theory

2nd Edition

MICHAEL V. SADOVSKII



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