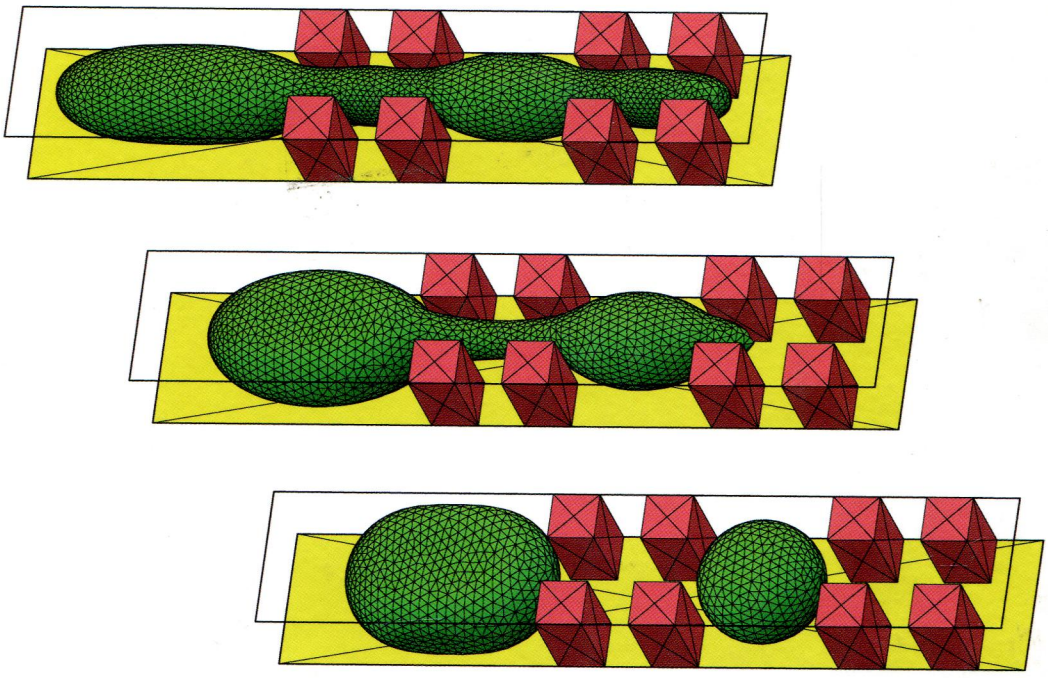
THE PHYSICS OF MICRODROPLETS



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The Physics of Microdroplets

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Co-published by John Wiley & Sons, Inc. Hoboken, New Jersey, and Scrivener Publishing LLC, Salem, Massachusetts. Published simultaneously in Canada.

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Cover design by Russell Richardson.

The cover shows a Surface Evolver simulation of a liquid plug in an enclosed rectangular hydrophobic channel, with square pillars added to constrict the channel (top). The pinched regions between the pillars have high curvature, hence high pressure, so liquid flows into the low-curvature, low-pressure larger blobs (middle), until the neck pinches off to form two stable blobs (bottom).

Library of Congress Cataloging-in-Publication Data:

ISBN 978-0-470-93880-5

Printed in the United States of America

10 9 8 7 6 5 4 3 2 1

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