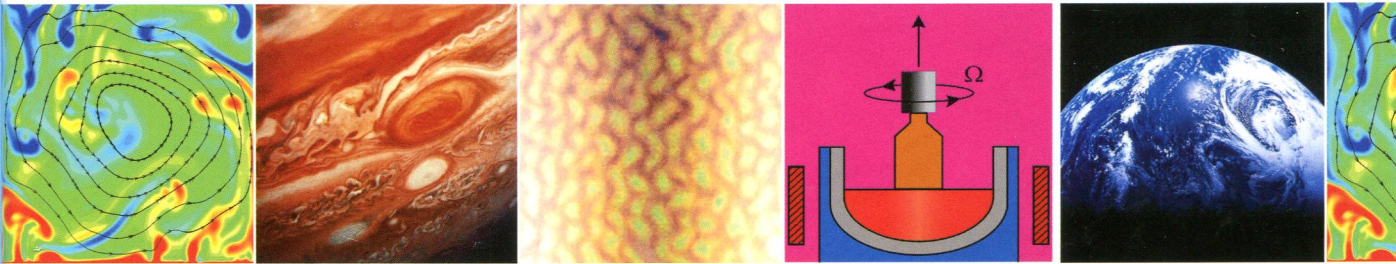


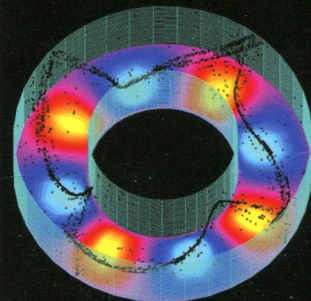
Marcello Lappa

Rotating Thermal Flows

in Natural
and Industrial
Processes



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Rotating Thermal Flows in Natural and Industrial Processes

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Naples, Italy



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