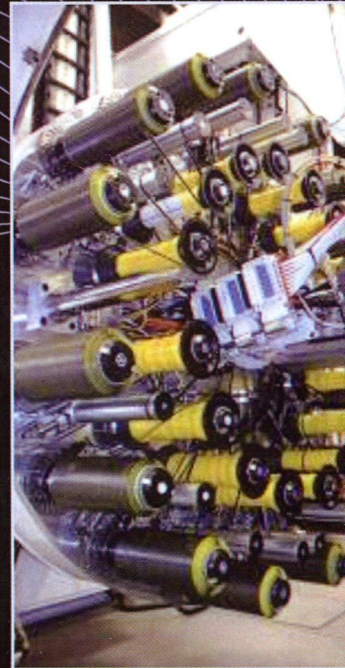
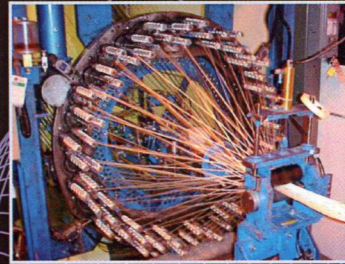


Structural Analysis of **POLYMERIC** Composite Materials

Second Edition

Mark E. Tuttle



CRC Press
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