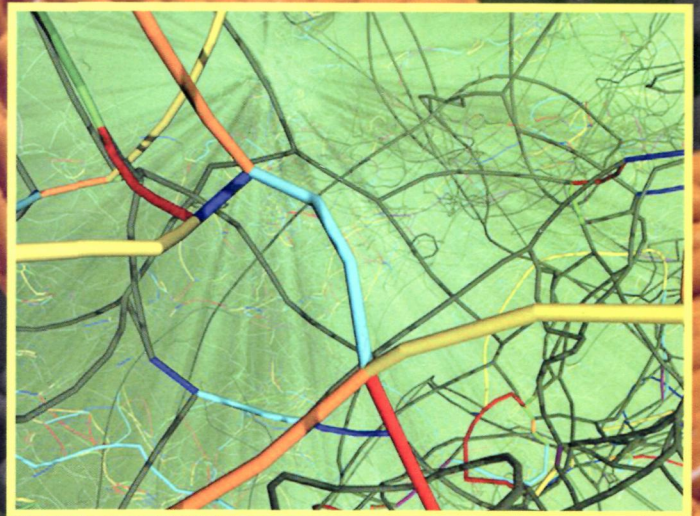


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Computational Modeling of Polymer Composites

A Study of Creep and
Environmental Effects



Samit Roy
J.N. Reddy

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